Measuring and modeling cosmic ray showers with an MBL system: An undergraduate project

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Following a brief historical introduction, a novel method for inducing and measuring cosmic ray showers using a low-cost microcomputer-based laboratory system is described. This reproduction of Bruno Rossi's classic experiment uses low counting-rate radiation monitors. The advantage of this is that a simple AND gate can be used to trigger coincidences, which makes the workings of the experiment completely transparent to undergraduate students. The disadvantage is that data must be taken for many days to get reasonably accurate results. A simple theory is presented that models the resulting shower curve quite well.
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I. INTRODUCTION

Modern day high-energy physics can be traced back to 1898 with the observation of the slow and unalterable discharge of an electroscope.[1–6] At that time, it was known that X rays and radioactive sources were both capable of discharging an electroscope; however, even when extreme care was taken to isolate the electroscope from all known sources of radiation, a slow discharge was always evident. In 1900, observations of this inexplicable discharge became the focus of serious investigations by Julius Elster and Hans Geitel in Germany and by Charles T. R. Wilson in the United States. Wilson was the first to suggest that this discharge might be caused by an extraterrestrial radiation of enormously high penetrating power. But after careful experiments revealed no significant difference between the discharge rates for an electroscope on the surface of the Earth and one in a remote railway tunnel, Wilson concluded that an extraterrestrial source was unlikely.

It was not until 1912, during a 6-h balloon ride to an altitude of over 5000 m, that Victor F. Hess performed an experiment that sparked an extraordinary series of investigations. The results of his experiment were best explained by assuming that a highly penetrating radiation enters the Earth's atmosphere from above.[7] At the time, this explanation was quite controversial, with Robert A. Millikan being one of the biggest critics. Ironically, Millikan's own experiments from 1922 to 1925 helped to confirm Hess's results and this actually led to Millikan getting much of the early credit for discovering the so-called cosmic rays. Still, it was Hess's 1912 balloon ride that marks the beginning of cosmic ray physics, for which he was awarded the Nobel prize in 1933.

Today we know that these *primary cosmic rays* are protons and bare nuclei of heavier elements of immensely high energies. While it is commonly believed that most galactic cosmic rays (those with energies below about 10^{16} eV) originate in supernovae, the source of ultrahigh energy particles ($\geqslant 10^{20}$ eV) is much less certain. Speculation ranges from galactic black-hole accretion disks, to gamma-ray bursts resulting from relativistic explosions, to the decay of topological defects that may have formed in the early universe.[8,9] While the origin of these particles is not directly relevant to this paper, a basic understanding of their interaction with the atmosphere will prove useful.

II. A COSMIC RAY PRIMER

When a primary cosmic ray particle collides with an atmospheric nucleus there results a number of *secondary* cosmic ray particles.[10] Consisting mainly of protons, neutrons, and pions, these secondary particles continue to collide with atmospheric nuclei, producing even more secondary particles. Apart from the production of pions, these nuclear collisions tend to result in more and more nucleons. For this reason, this portion of the secondary cosmic radiation is often called a *nucleon cascade*. Because of the large cross sections and energy losses typical in these collisions, these nuclear-active particles decay rapidly with atmospheric depth and only a small fraction survive to ground level.

The charged pions that are produced in the primary collisions have a lifetime of about 10^{-8} s before undergoing spontaneous decay into muons ($\pi^{\pm}\rightarrow\mu^{\pm}+\nu(\overline{\nu})$). These muons can also spontaneously decay ($\mu^{\pm}\rightarrow e^{\pm}+\nu+\overline{\nu}$), but due to their relatively long lifetime ($\sim 10^{-6}$ s) and high penetrating power, a large percentage typically survive to ground level. The component of secondary cosmic radiation consisting of charged pions and muons is often called a *meson shower*.

Finally, the neutral pions decay so rapidly into photons ($\pi^{0}\rightarrow 2\gamma$) that they are rarely involved in nuclear interactions. Through continual processes of pair production ($\gamma\rightarrow e^{+}+e^{-}$) and bremsstrahlung ($e^{\pm}\rightarrow e^{\pm}+\gamma$), these photons lead to what's called an *electromagnetic cascade*. While much less penetrating than the muons, a large percentage of these electrons, positrons, and photons also reach the ground.

The conglomeration of the various secondary components can produce literally millions of particles spread out over a region hundreds of meters in radius. Nevertheless, the basic situation is fairly simple. A single primary particle gives rise to three groups of secondary particles—the nucleon cascade, the meson shower, and the electromagnetic cascade. The nucleons decay so rapidly that they rarely reach the ground. The meson shower results in a large number of penetrating

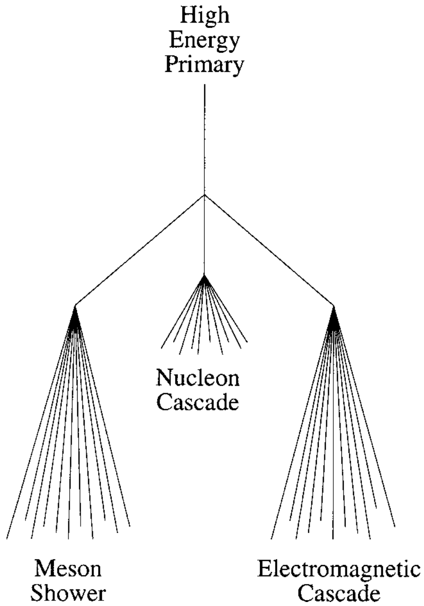

Fig. 1. A schematic diagram that shows the main features of the life of a typical primary cosmic ray particle. The primary particle gives rise to three groups of secondary particles, one of which is almost completely absorbed by the atmosphere. The particle groups are separated here for clarity.

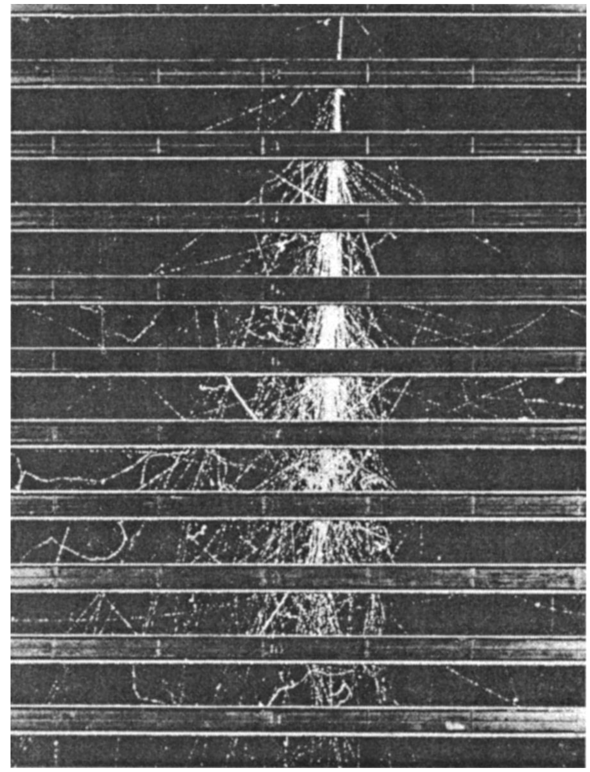

Fig. 2. A cloud chamber photograph (MIT cosmic ray group) of a particle traversing a series of brass plates demonstrates that a single particle can indeed give rise to multiple secondary particles. (Reproduced with permission of the McGraw–Hill Companies from Ref. 2, p. 96.)

muons, many of which reach the ground. And the electromagnetic cascade consists of electrons, positrons, and photons that are less penetrating than the muons, but many of which also reach the ground. This is sketched schematically in Fig. 1.

III. ROSSI'S EXPERIMENT

In 1929, before a distinction between primary and secondary particles was made, Dmitrii V. Skobeltzyn observed occasional cloud chamber tracks that appeared to show a single particle giving rise to multiple secondary particles. Using a slight variation of the coincidence counting technique developed by Walter Bothe and Werner Kohlhörster, Bruno Rossi confirmed Skobeltzyn's hypothesis with a simple and elegant experiment that was pivotal in helping physicists understand the nature of cosmic rays. He arranged an array of Geiger counters so that no single cosmic ray particle could trigger them all simultaneously. After covering this array with lead shielding, a large number of coincidences ($\sim$25 per hour) were recorded. Conversely, when the shielding was removed, the coincidence rate dropped to almost zero although still remaining higher than the expected ''accidental'' rate. Rossi reasoned that this was due to a single incoming particle giving rise to multiple secondary particles while traversing the lead (see Fig. 2). While this did not explain the coincidences that occurred with no shielding, it was soon realized that these were caused by showers being produced higher up in the atmosphere.

A plot of coincidence counts as a function of shielding thickness yields a ''shower transition curve'' that shows a sharp increase followed by a more gradual decrease (see Fig. 3). At the time, this shower curve was difficult to understand and it took years before a satisfactory explanation was developed. Qualitatively, the shape of this curve can be understand as follows. Showers are triggered by interactions between an incoming particle and a lead atom. Therefore, as the thickness of the lead shielding increases, the number of

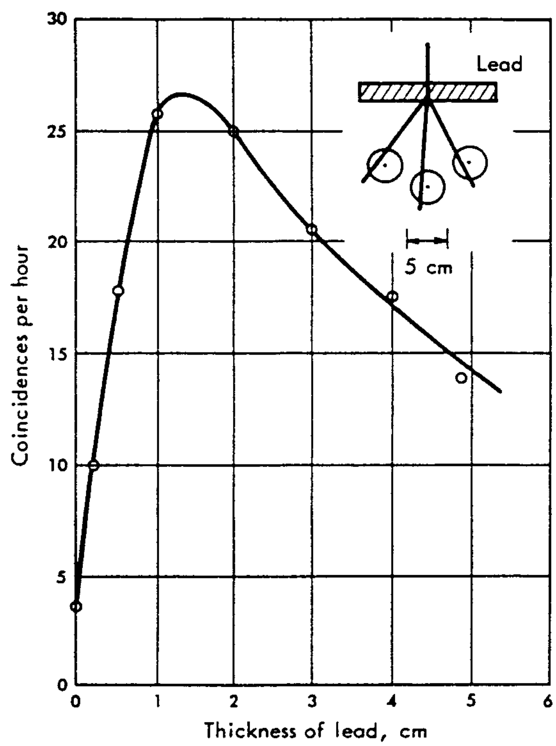

Fig. 3. Data obtained by Rossi in 1933. The inset shows a schematic diagram of a shower event that triggers multiple Geiger tubes in coincidence. (Reproduced with permission of the McGraw–Hill Companies from Ref. 2, p. 89.)

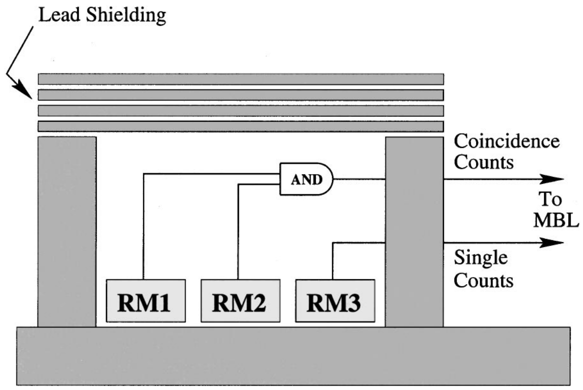

Fig. 4. A schematic diagram of the experimental setup. Two radiation monitors (RM1 and RM2) act as a coincidence counter and a third (RM3) keeps track of single counts. The experiment is housed in a lead structure that has a variable thickness roof.

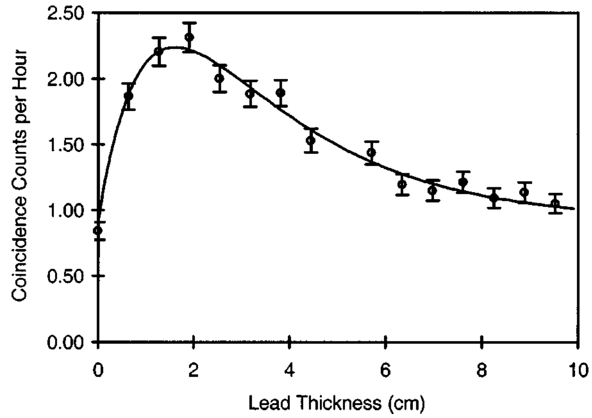

Fig. 5. Coincidence counts inside the lead house as a function of roof thickness. The error bars give the statistical error of the mean, $\sigma/\sqrt{N}$, and the solid curve is a fit using Eq. (5).

showers should also increase. This increased number of showers will result in a higher coincident counting rate as long as the shower fragments pass completely through the lead. But since the energy of the incident particle is divided between all of the shower fragments, it becomes increasingly likely that these fragments will be absorbed as the shielding thickness is increased further. Thus the increase in coincidence counts due to an increased number of showers is followed by a decrease in coincidence counts from absorption in the lead. This growth and decay behavior for a single shower can be seen clearly in Fig. 2.

IV. A MODERN ROSSI EXPERIMENT

Because of the increased power and decreased cost of personal computers, microcomputer-based laboratory (MBL) systems are becoming more and more popular in physics departments. An MBL system consists of a computer, an interface box, and sensors capable of measuring a wide range of physical properties including force, motion, light intensity, temperature, relative humidity, magnetic field, and radiation counts. One advantage of such a system is that the software interface is almost identical for every sensor. Thus, students who learn to use one sensor have effectively learned to use them all. This provides them with a powerful set of tools that they can use in a variety of experiments throughout the undergraduate curriculum.

An MBL coincidence counter. Aside from the lead shielding, the only piece of equipment needed to reproduce Rossi's experiment is a coincidence counter. A low-yield coincidence counter can be produced very easily using an MBL system and two radiation monitors. Since each monitor outputs an electronic pulse when it is triggered, a basic coincidence counter can be built by passing the output of these monitors through a simple AND gate. The AND gate will only pass a pulse to the computer when both monitors are triggered simultaneously. Covering the radiation monitors with a lead shield of varying thickness completes the experimental setup. A diagram of this is shown in Fig. 4.

Students will invariably ask about false counts that occur when two independent particles randomly strike the two monitors used in the coincidence counter at the same time. They should be challenged to determine the false counting rate by considering the probability that the two output pulses will ''overlap'' simply by chance. This accidental count rate is calculated by taking the fraction of time the two pulses can overlap in one period (from one monitor), and multiplying it by the count rate (of the other). For example, if the two monitors have count rates of N_1 and N_2 and the pulse width is τ, then the fraction of time the two pulses can overlap in one period is $2\tau/T_1 = 2\tau N_1$ and the accidental count rate is given by[11]

$$N_{\mathrm{acc}} = 2\tau N_1 N_2 . \tag{1}$$

For our hand-held radiation monitors with no lead shielding, the average count rate was $N_1 = N_2 = 519$ counts/h and the pulse width was measured to be $\tau = 135\,\mu$s. This gives a maximum accidental coincidence rate of 0.020 counts/h. This turns out to be negligible compared to statistical uncertainties and can be safely ignored in the final analysis. Nevertheless, students should be prepared to account for these accidental coincidences until their insignificance becomes obvious to them.

V. RESULTS AND ANALYSIS

Using a combination of bricks and $\frac{1}{4}$-in. thick sheets, a lead structure is built to house the coincidence counter. Students begin by testing the experiment overnight with a few centimeters of shielding to make sure everything is working properly. From this test, they are able to determine the approximate length of time needed to obtain reasonably good statistics. Measuring 24 coincidences over a 12-h session gives a statistical probable error of $1/\sqrt{24} \approx 20\%$. Thus, to achieve a probable error of less than 10% would require that data be taken for approximately 100 h, or about a week, for each lead thickness. This time commitment makes this experiment impractical as a week-long laboratory exercise, but it is quite effective as a semester-long student project. Figure 5 shows the results of the experiment after all the data have been collected. The characteristic sharp increase followed by a more gradual decrease is clearly visible, and students should be challenged to explain this behavior qualitatively.

A. Particle attenuation

During the weeks that data are being collected, the students can begin working on a simple theoretical model for the showering process. The first step is to consider the number of cosmic rays that actually enter the lead house as a

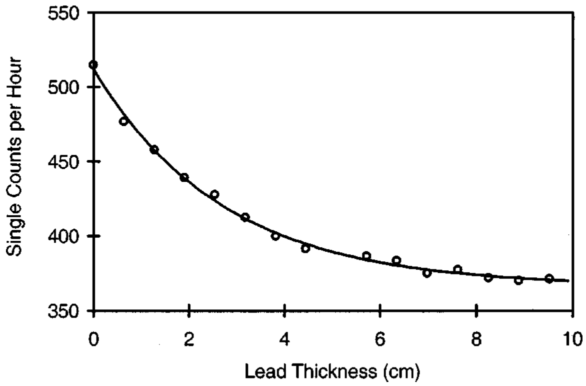

Fig. 6. Single counts inside the lead house as a function of roof thickness. As the solid curve demonstrates, the data are well approximated by Eq. (2).

function of shielding thickness. These are obtained by including a third radiation monitor inside the lead house. These single counts decrease and appear to level off to a nonzero value as shown in Fig. 6. The fact that these counts do not decay to zero has important implications that the students should be asked to explain. This behavior suggests that only a portion of the incoming cosmic rays are affected by the lead shielding. Thus, even if the students know nothing about the composition of cosmic radiation initially, these data force them to conclude that there is both a ''soft'' component that is strongly affected by the lead shielding, and a ''hard'' component that is relatively unaffected by the lead shielding. We call these the *interacting* and *noninteracting* components.

Students are quick to assert that the curve in Fig. 6 looks as if it decays exponentially. Thus, we can try to gain some quantitative information about these different cosmic ray components by fitting the data with a function of the form

$$N_{\mathrm{ni}}+N_i e^{-\mu x}. \tag{2}$$

The first term is independent of shielding and corresponds to the noninteracting particles, while the second term assumes that the interacting particles decay exponentially with shielding thickness. The absorption coefficient, μ, gives the probability per unit length that a single particle will be absorbed by the lead. A fit of Eq. (2) to the data yields $N_{\mathrm{ni}}=366$ counts/h, $N_i=146$ counts/h, and $\mu=0.368\,\mathrm{cm}^{-1}$ (see Fig. 6). The fact that this function fits the data so well allows us to draw two important conclusions. First, the noninteracting particles, presumably muons, make up about 70% ($N_{\mathrm{ni}}/N_{\mathrm{total}}$) of the cosmic ray particles at ground level. The students should check this result for consistency with published data.[12] And second, the interacting particles, presumably electrons, positrons, and photons, behave as if they were identical, monoenergetic particles. This is a bit of a surprise. One would expect these particles to have a distribution of energies, $N_i(\epsilon)$, and for the absorption coefficient to be energy dependent as well, $\mu(\epsilon)$. While this is undoubtedly true, the quality of the fit in Fig. 6 demonstrates that $N_i(\epsilon)$ and $\mu(\epsilon)$ conspire to produce the very simple behavior given by Eq. (2).

B. A simple model for showering

A satisfactory description of the showering process was developed in 1937 by Homi J. Bhabha and W. Heitler in England and simultaneously by J. F. Carlson and J. Robert Oppenheimer in the United States. The mathematical details relied heavily on the ''new Dirac quantum electrodynamics'' and took many years to work out.[13,14] Needless to say, this description is not very palatable for undergraduate students. Here, we develop a simple theory for the showering process that attempts to model this rather complex process in a manner accessible to undergraduates. To do so, we focus our attention on the two main features that produce the shape of the shower transition curve, absorption and showering. The assumptions that go into this model are as follows.

(1) The incident radiation is made up of interacting and noninteracting particles of which only the interacting particles can be absorbed or instigate shower events.
(2) Shower fragments consist of interacting particles emitted isotropically in the forward direction and any further showering of the fragments is considered part of the same shower and not counted separately.
(3) For thin thicknesses of lead, the probability that a single particle will be absorbed is proportional to the thickness of the lead.
(4) For thin thicknesses of lead, the probability that a single particle will shower is also proportional to the thickness of the lead but is independent of the absorption probability.
(5) A shower event is treated as a single unit in regards to absorption, meaning that the *entire* shower either survives or is absorbed.
(6) If a shower survives long enough to exit the lead shield, there is a fixed probability, P, that it will trigger the coincidence counter.

As mentioned above, the essential physics in these assumptions is a competition between a propensity for showering and absorption of the particles. In reality, these two processes are interrelated,[15] but it is mathematically simpler to treat them as distinct.

Let $N(x)$ represent the number of initial, interacting particles incident on the lead shield per unit time that have survived to a depth x. In a thickness dx, assumption (3) tells us that the particle flux lost to absorption is $\mu N(x)dx$ and assumption (4) tells us that the particle flux lost to showers is $\beta N(x)dx$. Here, μ is the absorption coefficient obtained from the data in Fig. 6 and β is an adjustable parameter (the showering coefficient) of the theory. The total flux of particles lost in a thickness dx is then given by $N(x)-N(x+dx)=(\mu+\beta)N(x)dx$, which can be solved to give

$$N(x)=[N_0 e^{-\beta x}]e^{-\mu x}, \tag{3}$$

where N_0 is the initial particle flux at $x=0$. As already stated, Eq. (3) gives the initial particle flux that has survived to a depth x in the lead shield. But we are interested in the *shower flux*—the number of shower events per unit time—that survives to a depth x in the lead shield. To determine this, notice that the quantity in square brackets in Eq. (3) represents the initial flux that has *not* undergone a showering transformation, and the exponential factor, $e^{-\mu x}$, accounts for absorption. Thus, we need only replace the initial particle flux that has *not* produced showers, with the initial particle flux that *has* produced showers to get

$$N_s(x)=[N_0(1-e^{-\beta x})]e^{-\mu x}. \tag{4}$$

Equation (4) gives the shower flux that survives to a depth x in the lead shield. Before it can be compared to experiment, there are two issues that need to be resolved. The first is the probability of actually measuring one of these showers. This will depend, among other things, on the size, sensitivity, and placement of the radiation monitors, and also on the details of the shower event (e.g., the number of particles and their angular distribution). Using assumption (6) above, we simply assume that any shower event that emerges from the lead shield will trigger the detector with a fixed probability, P.

The second issue is the effect of atmospheric showers. Obvious to students is the fact that the data in Fig. 5 show a nonzero number of coincidences when there is no shielding that is much higher than the accidental rate calculated earlier. Clearly, these coincidences cannot be caused by shower events occurring in the lead shield. Furthermore, as the shielding thickness is increased beyond about 5 cm, the coincidence counts appear to level off to the no-shield value. Therefore, whatever is causing these coincidences is not affected by the lead shielding. This suggests that these coincidences are caused by the noninteracting particles. Recall that about 70% of the cosmic radiation that survives to ground level consists of muons. Since these muons are created as part of the meson shower, it is not unreasonable to expect that some of these showers will trigger coincidence counts that are unaffected by the lead shielding. These atmospheric muon showers can be accounted for by adding an overall constant, N_{as}, to Eq. (4).

Our final prediction for the measured number of coincidence counts per unit time is then

$$N_{\mathrm{cc}}=N_{\mathrm{as}}+PN_0(1-e^{-\beta x})e^{-\mu x}. \tag{5}$$

In this equation, N_{as} is determined from the zero thickness coincidence counts in Fig. 5, μ is determined from fitting Eq. (2) to the single count data, and N_0 is determined by multiplying the incident particle flux measured by one radiation monitor (N_i) by the ratio of the shielding area to the cross-sectional area of a G–M tube inside the monitor. Thus, there are only two free parameters in this simple model, β and P. Fitting Eq. (5) to the experimental data yields $\beta=0.602\,\mathrm{cm}^{-1}$ and $P=2.13\times10^{-4}$.[16] Considering the simplicity of the model used to describe this complicated process, the fit is quite good (see Fig. 5).

The fit parameters from Eqs. (2) and (5) tell us that the showering coefficient (β) is larger than the absorption coefficient (μ). This means that, within the context of this model, a primary particle is about 1.6 times more likely to produce a shower than to be absorbed as it travels through the lead shield. It should also be noted that the low probability of triggering the coincidence counter (about a 0.02% chance) should not be too surprising. An order of magnitude estimate of this can be made by calculating the probability that two random particles entering the lead house simultaneously will actually trigger the two radiation monitors. In our experiment, this comes out to 0.013%, which is certainly the right order of magnitude. It is this inefficiency at detecting showers that results in such long experiment times. By moving the radiation monitors closer together, one might be able to increase the shower detection efficiency, and possibly learn something about the angular distribution of the shower fragments as well.

VI. CONCLUSION

This project contains a good mix of experiment and theory that will challenge even the best undergraduate students. By using an MBL system, the experiment itself is very easy for students to understand, although it lengthens the time needed to perform it considerably. This makes a nice, semester-long project for a group of two or three students, particularly if they are asked to develop the mathematical model on their own (with assistance from the instructor as needed). Besides learning the basics of cosmic ray physics, students must manage a lengthy experiment that involves a fairly large amount of data collection and analysis. In addition, this project provides a good opportunity to engage students in the process of mathematical modeling, something few undergraduates get much practice with. This combination of theory and experiment results in a fairly extensive overall learning experience for the students.

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[10]The interactions involved in these collisions are quite complicated. Here, we are only interested in the most notable features.

[11]Note that we are assuming *any* overlap will trigger the AND gate and are thus estimating a worst-case scenario. In actual practice, the AND gate may not trigger if the overlap is extremely small.

[12]Most sources quote between 70% and 75% as the muon component of cosmic radiation at sea level. See, for example, Particle Data Group, Eur. Phys. J. C **15**, 151–152 (2000).

[13]An overview is given in Ref. 1, pp. 202–225.

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[16]Note that N_{as} is only determined experimentally to lie within a small range, $N_{\mathrm{as}}=0.84\pm0.07$. Adjusting N_{as} within this range to obtain the best fit yields $N_{\mathrm{as}}=0.91$.