On Using a Barter Edgeworth Box to Discuss Efficiency Early in the Semester

(*Note: this file includes materials from the Excel file for ease of reference as Appendices***)**

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URL, Faculty Version

http://users.dickinson.edu/~erfle/ExcelGraphics/BEBInstructor.xlsx

URL, Student Version (with answers and instructor notes removed):

http://users.dickinson.edu/~erfle/ExcelGraphics/BarterEdgeworthBoxHW.xlsx

300 word descriptive note for the **Online Section** of *The Journal of Economic Education*

A cornerstone of economic analysis is the Pareto criterion for economic efficiency. Introductory and intermediate microeconomics texts discuss Pareto (economic) efficiency very early in the text but relegate a formal treatment to near the end of the text.

The connection between distributive efficiency and exchange can be brought into closer relief by employing a barter Edgeworth box as soon as students have been introduced to preferences and their graphical representation as indifference curves, even before students learn about utility functions or budget constraints. Bringing this discussion forward creates a quick linkage between their childhood intuition that trading with their peers often makes them better off and how economists model such trades as Pareto superior redistributions of goods. The Edgeworth box elegantly illustrates the power of using simplifying assumptions (two individuals, two goods) to build an economic model.

The story presented is readily remembered. Most students recall going trick-or-treating and trading afterward. Annie ends with lots of gummis, Bob has lots of chocolate. They will likely trade so that each achieves a more balanced distribution of candy.

The BarterEdgeworthBox.xlsx file allows you to build the box and model trading in class via computer projection without having to spend class time drawing the box. Separate sheets show this analysis with two separate graphs and, more elegantly, as an Edgeworth box. Each interactive graph allows the user to add or remove axes, individual curves, and change allocations and relative bargaining strength via sliders. The file includes a separate introduction sheet with static graphs and questions, glossary, answer key and instructor notes. This file is appropriate for introductory classes using indifference curves and all intermediate classes. Although this can be used on its own, it works best if it is introduced as part of a lecture prior to student distribution for exploration or homework.

__Note:__ Explanation of JEE's Online Section (from JEE website).

The primary goal of the Online section of *The Journal of Economic Education* is to identify exemplary material for teaching and learning economics that is interactive or otherwise not conducive to traditional printed-page format. It provides a timely outlet for noncommercial work by economists and educators who are creating teaching materials using innovative electronic technology.

Those submitting material to be reviewed in the Online section must e-mail a potentially publishable descriptive note of at most 300 words to W. B. Walstad, Editor, *The Journal of Economic Education*, at jee@unl.edu . The note must describe how the submitted site extends the pedagogical and/or technological frontiers and contributes to teaching and learning economics. Notes describing material at sites that are reviewed positively will be published in the *JEE*.

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Appendices

(*Screenshots from material available at the URL listed above*)

These appendices do not show all sheets in the file but all that would be used if a faculty shows the file in class prior to distribution (as planned).

You should follow different directions based on WHO YOU ARE

1. **Instructors** using this for the first time. Read the note starting in row 9 then use as you wish.
2. **Instructors** who has already used this file in class. Delete the **InstructorNotes** & **Answers** sheets prior to distribution.
3. Stand-alone user. You do not need to read the Instructor Notes below which are written for academic economists. Start on the **Introduction** sheet and make sure you read the **Story** pdf before attempting to answer questions. You can check your work on the Answers sheet.
    - I expect stand-alone users to be able to do Questions 1-11 on the **Introduction** sheet.
    - Working through the file and answering questions on your own may well take a couple of hours.
    - Don't worry if you don't know Excel as long as you have Excel on your machine. This file runs on Excel but all you need to do is use sliders and click-boxes to use the file.

INSTRUCTOR NOTES

This file works best if you work through building the Box in class using computer projection.

- If you build up the box in class, most students feel comfortable using just the **Introduction** and the **EdgeworthBox_(fig.6-8)** sheets.
    - These sheets have yellow tabs. The others are supplementary support sheets.
    - Most sheets start with numbers. These numbers refer to figures from the **Story** pdf.
- I have used this file for both Introductory and Intermediate Microeconomics classes.
    - I find that the in-class discussion can be done in about 20 minutes in an introductory class and 15 minues in an intermediate class.
        - Questions 1 - 11 are appropriate for introductory microeconomics.
        - All questions are appropriate for intermediate classes.
    - In both classes, I do this as soon as I talk about indifference curves.
        - I do not wait until late in the semester to use it.
        - I even do it before talking about utility functions and budget constraints.

I have found that for an undergraduate audience, this is an easy sell. They remember going trick-or-treating and they also remember the trading that occurs at the end of the night.

- In many ways this is an easier story to get across to students than Production Possibility Frontiers.
    - Most texts have a discussion of Pareto efficiency very early in the book but none of the intermediate texts I've seen show an Edgeworth Box prior to 2/3 of the way through the text.
        - I believe that this misses a great opportunity to solidify some basic consumer optimization rules and at the same time have a basic discussion of why people trade with one another.

One of the reasons to show the file in class prior to giving to students as homework is that students then do not need to worry about learning how to use the **Scenarios** function in order to navigate the file.

- It is particularly enlightening to show students how the Box is built by adding both endowments.
    - Do this using the **2-4,TwoGraphBarter** sheet.
    - You can show how to build the Box before talking about Pareto improvements using this sheet but I think that it works best to show the adjustment process using the **2-4, TwoGraphBarter** sheet prior to showing the Box.
        - That way they see how cumbersome it is to look at multiple individual graphs.
    - To show how the Box is built, click the two Endowment as vector boxes in E26 and M26.

        - Then click the box in A30 to show how Bob's endowment is added to Annie's to get total resources available.

            From here, click the box in I30 to show the Box.

- Next, I spend a minute or two on **5Allocations**.
    - The point here is so simple that students get it almost immediately.
    - Nonetheless, they need to be told this is true. After all, they now have to think upside down.
        - Use the clickbox in A19 to show the punchline.
        - Then go to the main sheet, **6-8,EdgeworthBarter**.

I typically start with nothing clicked on then I simply build the Box as I talk about it in class.

- You can unclick all boxes yourself of use the **Blank** scenario on the **Data** sheet.

We've all done this descriptive analysis as we struggle to do things upside down with chalk or markers.

- By providing students with a figure they can move themselves, they are able to focus on what it means to have unequal MRSs and how we can achieve multiple Pareto optimal outcomes based on differences in bargaining ability.
    - Two trade ratios are examined: 9:10 (the ratio of total Y:X) is the main ratio used.
        - Of course, this is what the price version will end up using in this instance if we were to formally introduce budget constraints.

            This is depicted in Figures 3-4 and 6-7.
        - The other trade ratio discussed in the **Story** is 1:1. This is probably what I would have done, had I been Bob.

            This is depicted in Figure 8.

If handing this out to students, delete the InstructorNotes and Answers sheets and have the students start from the Introduction sheet, cell A1.

BACKGROUND

Individuals trade because each is made better off in the process.

- Economists say that such trading leads to a ***Pareto superior*** reallocation of resources. In the consumer context, the resources being redistributed are goods.
  - Individuals stop trading when Pareto superior redistributions are no longer available.
    - This stopping point is called a ***Pareto optimal*** distribution of goods.
- One can examine these issues using the consumer version of an Edgeworth box.

An Edgeworth box provides an economic view on why individuals gain from trading with one another.

- This version of the Edgeworth Box does not even require a price system to work.
  - All that is required is you have a basic understanding of ***preferences*** and their geometric representation as ***indifference curves***. The slope of an indifference curve is the rate that the consumer is willing to trade one good for another. This is called the marginal rate of substitution or **MRS**.
- If you feel uncomfortable with the concept of preferences, indifference curves, or MRS, click on the **Preferences** pdf.

Preferences.pdf

This file examines efficiency in the context of a barter version of the Edgeworth box.

If you have already seen the Edgeworth box presented in class, click here or go directly to the main sheet, **EdgeworthBarter_(fig.6-8)**.

- If you want to just see the basic story setup and the assumptions for this model, click on the **Assumptions** pdf.

Assumptions.pdf

Note: Sheets used to create figures from the **Story** pdf end with _(fig.#)

- If you have not seen the Edgeworth box built in class or if you wish to review what your professor described in class in building the Edgeworth box model you should at least skim the **Story** pdf.
- The **Story** pdf includes discussion of the assumptions of the model as well as the two graph analysis and how both graphs are incorporated in to the Edgeworth box.

Story.pdf

QUESTIONS

The questions below require you to at least skim the **Assumptions** pdf.

Given Assumptions 1-3 (**Assumptions** pdf or the first page of the **Story** pdf), would Annie and Bob trade with one another if they ended up with the following in their bags? If so, provide bounds on their trade ratio of y per x.

1 Annie	x = 10	and	y = 20	Bob:	x = 15	and	y = 15
2 Annie	x = 20	and	y = 15	Bob:	x = 15	and	y = 15
3 Annie	x = 15	and	y = 20	Bob:	x = 25	and	y = 20
4 Annie	x = 20	and	y = 15	Bob:	x = 28	and	y = 21

For questions 5-8, assume that Annie's and Bob's parents agree to only allow their children to trade at a rate of 1 y per x. In this instance, will trade occur? If so, explain who will get more x and who will get more y. If not, explain why not.

5 Based on the endowments in Question 1.

6 Based on the endowments in Question 2.

7 Based on the endowments in Question 3.

8 Based on the endowments in Question 4.

9 What MRS results in the Pareto optimal solution given:

A) Total candy is based on Question 1.

B) Total candy is based on Question 2.

C) Total candy is based on Question 3.

D) Total candy is based on Question 4.

Hint: The Parto optimal MRS in the ***Story*** *pdf was 0.90. What was true about the ratio of total y to total x in that story?*

The **Story** pdf argues that the trade 38 for 38 is a Pareto optimal solution in Annie and Bob's barter economy (**Figure 8**). Other possible Pareto optimal solutions exist and the one chosen will depend on Annie's and Bob's relative bargaining strength. Use the sliders on the **EdgeworthBarter_(fig.6-8)** sheet to answer questions 10 and 11.

10 Suppose Annie dominates the bargaining completely and leaves Bob just as well off as he was at his initial endowment. How much y will Annie trade to get more x. What is the y per x trade ratio in this instance? What bundle is this in the Edgeworth Box in **Figure 7B** of the **Story** pdf?

11 Suppose Bob dominates the bargaining completely and leaves Annie just as well off as she was at her initial endowment. How much x will Bob trade to get more y. What is the y per x trade ratio in this instance? What bundle is this in the Edgeworth Box in **Figure 7B** of the **Story** pdf?

12 If Annie and Bob had to maintain the 1 for 1 trades required by their parents but could do this a bit at a time rather than as a "take it or leave it" proposition, would 38 for 38 have been a solution? If not, explain who would stop trading first, and at what level of overall trading.

Hint: To answer this question, you must consider separately where Annie and Bob would wish to stop given a trading ratio of 1 y per x. What is MRS at this point?

13 Given a trade ratio of 1 for 1, which of the goods has "excess demand" and which has "excess supply" given Annie and Bob's preferences? Your answers to question 12 should help you here.

14 Is the result obtained in question 12 Pareto optimal? If not, explain the bounds on the trade ratio that exist in this instance. *Hint:* You can answer this using the sliders on **EdgeworthBarter_(fig.6-8)** sheet.

The rest of the questions are a bit more challenging. Some require mathematical proofs.

15 Show that the set of Pareto optimal distributions in the barter model developed in the **Story** pdf is the set of points $y_A = 0.9 \cdot x_A$.

Hint : If Annie has x_A units of x and y_A units of y, how much of each good does Bob have?

16 Explain why each of the Pareto optimal outcomes in the barter model developed in the **Story** pdf had $MRS_A = MRS_B = 0.9$. (This is easy to do if you have already done question 15.)

17 Prove that the Pareto optimal solution depicted in **Figure 8** is close to, but not exactly on the contract curve. The true answer is involves fractional trades rather than whole number trades. What are those fractions?

18 Replicate the analysis in the **Story** pdf using a new scenario. Suppose initial endowments are:

Annie x = 40 and y = 140 Bob: x = 60 and y = 60

Before trading begins, Annie and Bob's parents look at the piles and say to Annie and Bob: You can trade with each other, but given how many gummis there are relative to chocolate, we suggest trading 2 gummis per chocolate.

a. What Pareto optimal distribution of goods results and what are their MRSs?
b. Who is better off in the end and why?

Hint: It may help to draw your own Edgeworth Box.

Background on Preferences, Indifference curves, and MRS

Preferences are an individual's valuation of goods and services independent of budget and price.

- Preferences are the starting point for understanding consumer behavior.
- If you are offered (without charge) two bundles of goods (think shopping carts) and you are asked:
  - *Do you prefer **A** or **B** or are you indifferent between the two?*
    - You will always answer that question one of three ways: 1) you prefer **A** to **B**,
2) you prefer **B** to **A**,
or 3) you are indifferent between the two.
    - In symbols we would say either **A** $\succ$ **B**; **B** $\succ$ **A**; or **A** ~ **B**.

An **indifference curve** is the set of bundles that provide the same satisfaction and between which the consumer is indifferent.

- Indifference curves provide a geometric representation of preferences.
- Preferences are said to be **well-behaved** if they are *monotonic* and *convex* .
  - **Monotonicity** simply means that more of a good is at least as good as less of that good.
  - Strict monotonicity simply means that more is better than less -- or, goods are good!
    - Strict monotonicity means indifference curves slope downward.
    - Strict monotonicity precludes perfect complements.
  - **Convexity** means that an individual likes a variety of goods rather than one good to the exclusion of all others.
  - (The formal definition of convexity is that the set of bundles that are at least as good as an initial bundle is a convex set.)
    - Strict convexity requires that averages are strictly preferred to extremes.
    - This implies that indifference curves cannot be linear or have linear parts.
    - Strict convexity precludes perfect substitutes.

If we require monotonicity and strict convexity, then preferences are really well-behaved.

- Really well-behaved indifference curves are downward sloping and the slope declines as more x and less y is consumed.
  - This is most readily described using MRS.

The **Marginal Rate of Substitution** is defined as MRS = **-** $\Delta y/\Delta x$ along an indifference curve.

- MRS represents the rate at which an individual is willing to exchange y for x in order to remain equally happy.
  - Notice that MRS is negative the slope, but given monotonicity, the slope is negative so MRS is positive.
- MRS is the rate at which the individual is willing to substitute y for x along an indifference curve.
  - This rate should depend on how much x and y the individual has.
    - Indeed, if preferences are strictly convex, MRS continually changes (or the indifference curve really is a curve).

Monotonicity and strict convexity implies that MRS declines along the downward sloping indifference curve as x increases.

Applying the preference model: A simple model of barter

Even without utility functions and budget constraints we can examine some of the power of preferences by examining preferences in the face of a **barter** economy.

Definition: **Barter** involves exchanging goods and services without the use of money.

Before you understood money or had an allowance you probably still traded things with your friends – that is, you knew how to redistribute goods to make yourself better off. When you went to school with a ham and cheese sandwich and your friend had peanut butter and jelly, you sometimes traded sandwiches because both of you were better off as a result of the trade. Or, after a long evening of trick-or-treating you may well have spread out your haul on the living room floor and exchanged candy with your brother, sister, or friend. These are extremely simple examples of redistributing goods via barter. The model of preferences allows you to analyze such exchanges. Initially we will do this with two separate diagrams, then we will redo the analysis using an elegant and powerful tool called the Edgeworth Box.

The model we develop is admittedly simplistic, but will give you the flavor of things to come. The following assumptions are sufficient to describe how barter can lead to superior outcomes. Annie and Bob are two friends who go trick-or-treating on Halloween. They only care about chocolate, x, and chewy candy (like gummi bears and jelly beans), y. (Since the analysis is fairly complex we will sometimes revert to x and y as labels rather than chocolate and gummi bears.) At the end of the evening they find that Annie has lots of gummis, and Bob has lots of chocolate. They have the following in their bags:

Annie has $\mathbf{(x_A, y_A) = (40,108)}$ Bob has $\mathbf{(x_B, y_B) = (110,27)}$.

Economists say these bundles represent their *initial endowments of goods*.

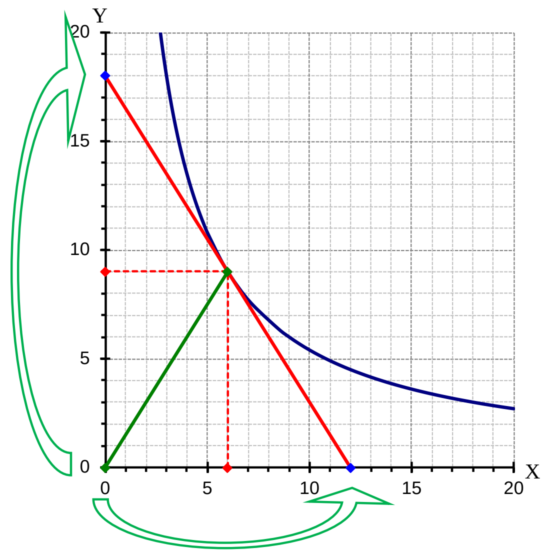
Both would like a more balanced haul to take home, so they are certainly going to benefit from trading with each other. To describe their situation more completely we need to know more about their preferences.

Assumption 1. Annie and Bob have identical well-behaved (monotonic and strictly convex) preferences over (x,y) bundles.

Assumption 2. They like x and y equally well in the following sense: If they have more y than x, they would be willing to give up more than 1 y for 1 more x and visa-versa. Only if $x_o = y_o$ would they be willing to trade 1:1. In particular, their MRS at any bundle $(x_o,y_o) = y_o/x_o$. (This means that the slope of their indifference curve at any bundle, (x_o,y_o), is $- y_o/x_o$.)

Assumption 3. They are willing to exchange non-integer quantities of candy if need be. After all, they know that there are lots of M&Ms in a pack: the same goes for gummi bears and Dots.

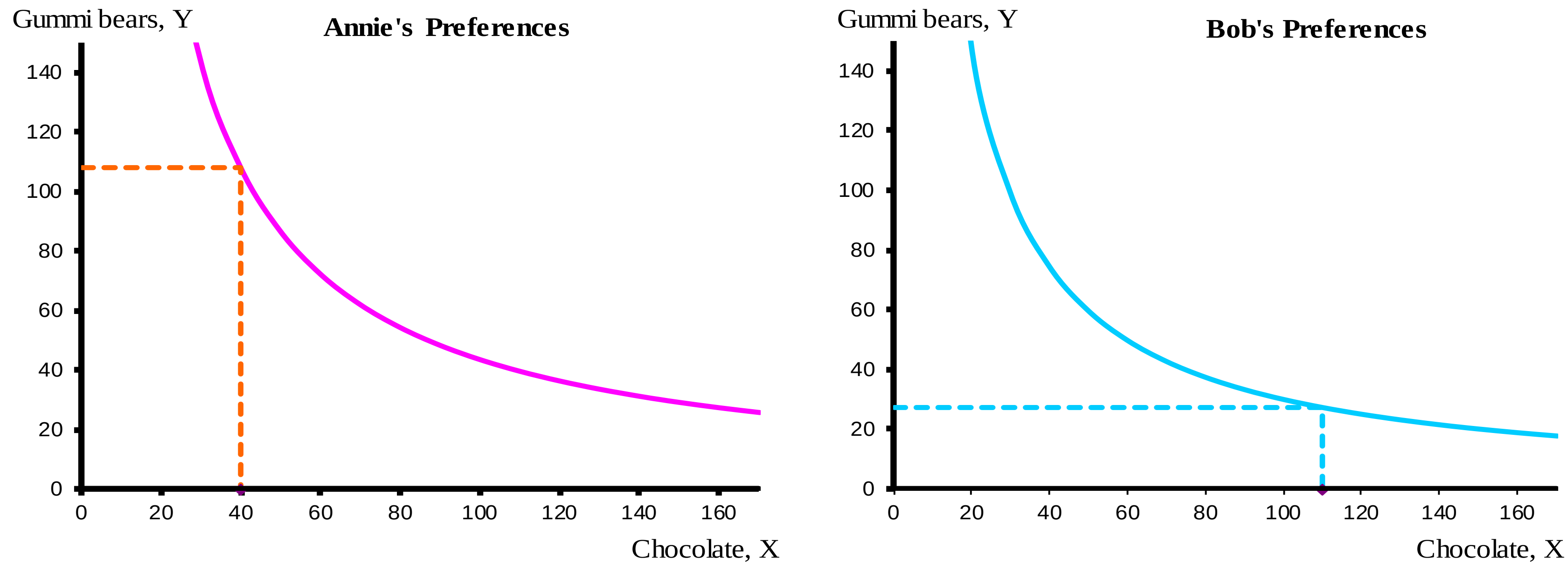
Figure 1 (visualizing Assumption 2: MRS = y/x)



Assumption 2 is difficult to see in the figures since the grid system has been suppressed to reduce visual clutter. It turns out that this particular set of preferences is used a lot in microeconomics. There is an easy geometric way to see what indifference curves look like if MRS(x,y) = y/x. **Figure 1** shows the bundle (6,9) and the blue indifference curve through this bundle satisfies Assumption 2. The red tangent line is created using a simple geometric trick. Double the x coordinate along the x axis (from 6 to 12) and double the y coordinate along the y axis (from 9 to 18) and connect these endpoints, the result is the red tangent line with the tangent bundle right in the middle. One way to think of this is to imagine flipping the origin bundle out to the point (12,0) using the x = 6 vertical line as the axis of rotation. The green ray would rotate out to the lower part of the red line (this is what the green arrow below the x axis signifies). A symmetric rotation using the y = 9 horizontal line as the axis of rotation would rotate the green ray out to the upper part of the red line (the green arrow to the left of the vertical axis shows this rotation).

Assumption 3 allows us to treat their preferences as continuous rather than discrete. As a result we can draw an indifference curve through each of their endowment points. This is done in **Panel 2A** and **2B** (*BarterEdgeworthBox.xlsx* created all figures).

Panel A **Figure 2** **Panel B**



Annie's
☑ Endowment
☑ Initial indifference curve

☑ Initial Endowment		MRS
X	Y	(= Y/X)
40	108	2.70

Bob's
☑ Endowment
☑ Initial indifference curve

☑ Initial Endowment		MRS
X	Y	(= Y/X)
110	27	0.25

Annie's MRS of gummis for chocolate is $MRS^A = 2.70$ at her endowment point. She would be willing to give up more than two and two thirds gummi packages for one package of chocolate. By contrast, Bob's MRS of gummis for chocolate is $MRS^B = 0.25$ at his endowment point. He would be willing to give up a quareter of a package of gummis for another package of chocolate. Or, viewed in terms of giving up chocolate, he would be better off as long as he gets at least one fourth of a pack of gummis. This is not surprising given how much chocolate he already has. Given these MRSs, an exchange of one package of Bob's chocolate in exchange for E packages of Annie's gummis will make both better off as long as $MRS^B = 0.25 < E < 2.70 = MRS^A$. Economists say that such exchanges represent **Pareto superior** redistributions of goods.

Definition: A redistribution of goods is **Pareto superior** to the initial distribution if no one involved is made worse off and at least one is made better off.

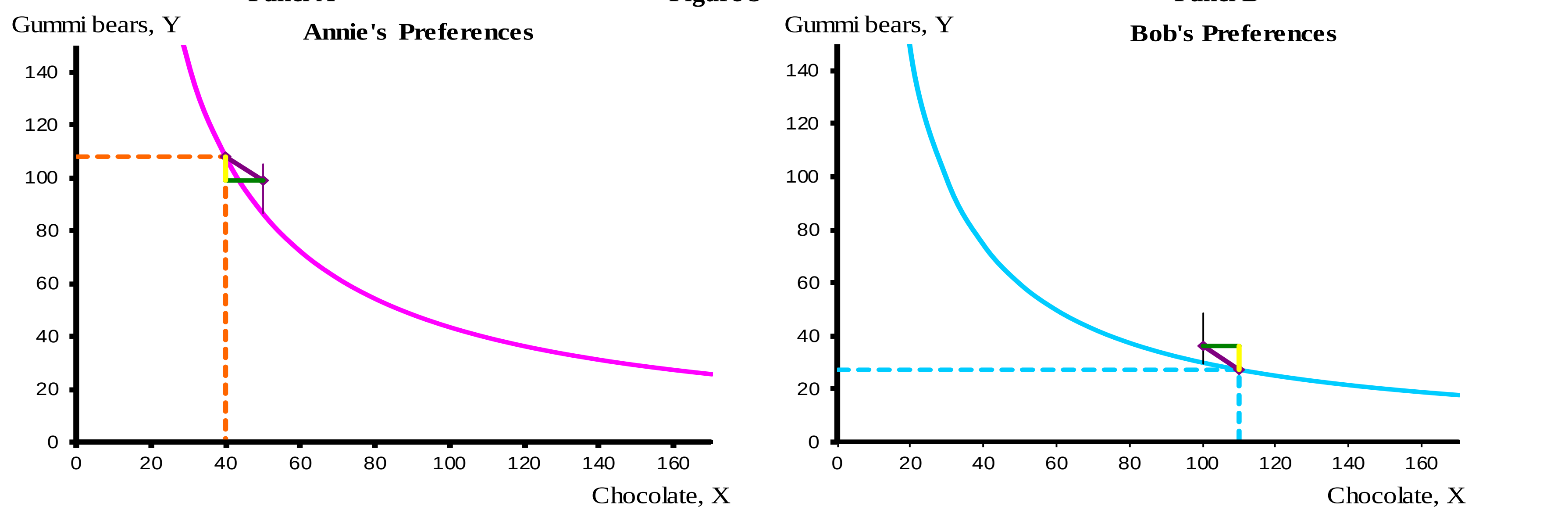
Exchange will be mutually beneficial as long as their MRSs are not equal to one another. Once Annie and Bob have equal MRSs there is no longer a possibility of mutually beneficial exchange: they cannot <u>both</u> be made happier. Economists say that such distributions are **Pareto optimal**.

Definition: A distribution of goods is **Pareto optimal** if there are NO redistributions of goods in which least one is made better off and no one involved is made worse off.

Alternatively we can define Pareto optimality in terms of Pareto superiority. A distribution is Pareto optimal if there are no Pareto superior redistributions.

Considering these exchanges one package at a time would take a lot longer than Annie and Bob want to devote to this project. As a result, Annie and Bob initially consider a trade of 10 chocolates. Figure 3 depicts this situation. Annie would be willing to give up about 27 packs of gummis for 10 chocolates since Δy = slope*Δx (recall slope = $\Delta y/\Delta x$ so we obtain an approximation of how much y changes on an indifference curve as x changes by multiplying slope times how much x changes). Bob would be willing to exchange 2.5 packs of gummis for 10 chocolates for the same reason. A mutually beneficial exchange occurs somewhere between 2.5 and 27. Each of these numbers is an approximation since the indifference curves are CURVES. An exact answer for the maximum number gummi packages Annie would be willing to give up for 10 more chocolates would be based on finding the point on Annie's initial indifference curve associated with 50 chocolates (her initial 40 plus 10 from

Panel A **Figure 3** **Panel B**



Annie's
- ☑ Endowment
- ☑ Initial indifference curve
- ☑ Barter choice
- ☑ Barter possibilities

☑ Initial Endowment		MRS
X	Y	(= Y/X)
40	108	2.70
☑ Annie's Barter bundle		
50.0	99.0	1.98

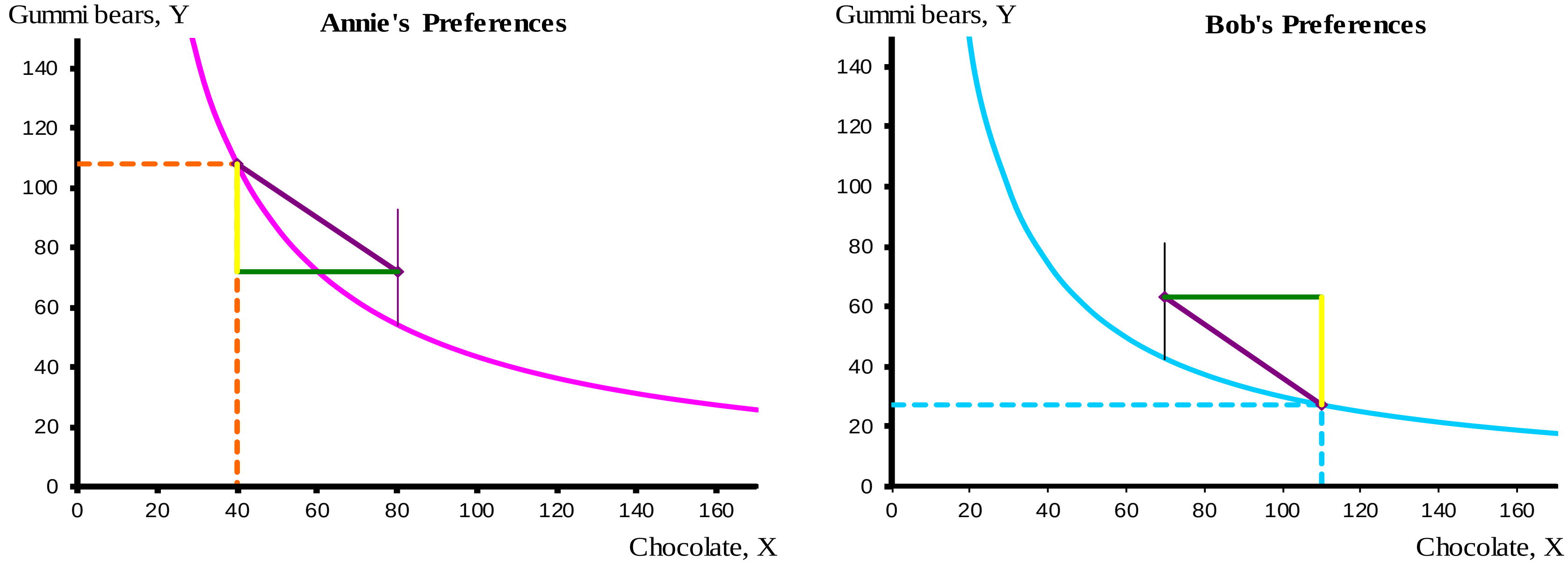
Bob's
- ☑ Endowment
- ☑ Initial indifference curve
- ☑ Barter choice
- ☑ Barter possibilities

☑ Initial Endowment		MRS
X	Y	(= Y/X)
110	27	0.25
☑ Bob's barter bundle		
100.0	36.0	0.36

Bob). In fact, (40,108) ~ (50,86.4) so Annie would be willing to give up at most 21.6 gummis for 10 chocolates (21.6 = 108-86.4). Bob's actual minimum required rate of exchange for 10 chocolates is 2.7 gummis. Redistributions of goods where Bob gives Annie 10 chocolates in exchange for E gummis with $2.7 \leq E \leq 21.6$ are Pareto superior to the initial distribution. The solid vertical line in each panel of **Figure 3** represents the set of Pareto superior redistributions in this instance. **The purple diagonal line shows one such trade, 9 gummis for 10 chocolates.** The green horizontal segment represents the chocolate traded and the yellow vertical segment represents the gummis traded. The **TwoGraphBarter_(fig.2-4)** sheet allows you to vary each of these values to create your own Pareto superior redistribution.

The 9 for 10 trade is not the end of the story however, because Annie's and Bob's MRSs are still not equal ($MRS^A = 1.98$ at (50,99) and $MRS^B = 0.36$ at (100,36)). The exchange has reduced the gap between their MRSs, but it has not eliminated it. If Annie and Bob continue to exchange gummis for chocolate at a 9:10 rate, they will achieve a Pareto optimal distribution in which Annie has (80,72) and Bob has (70,63). The MRS of this distribution is 0.90 for both children. **Figure 4** provides the Pareto optimal distribution described above.

Panel A **Figure 4** **Panel B**



Annie's
- ☑ Endowment
- ☑ Initial indifference curve
- ☑ Barter choice
- ☑ Barter possibilities

☑ Initial Endowment		MRS
X	Y	(= Y/X)
40	108	2.70
☑ Annie's Barter bundle		
80.0	72.0	0.90

Bob's
- ☑ Endowment
- ☑ Initial indifference curve
- ☑ Barter choice
- ☑ Barter possibilities

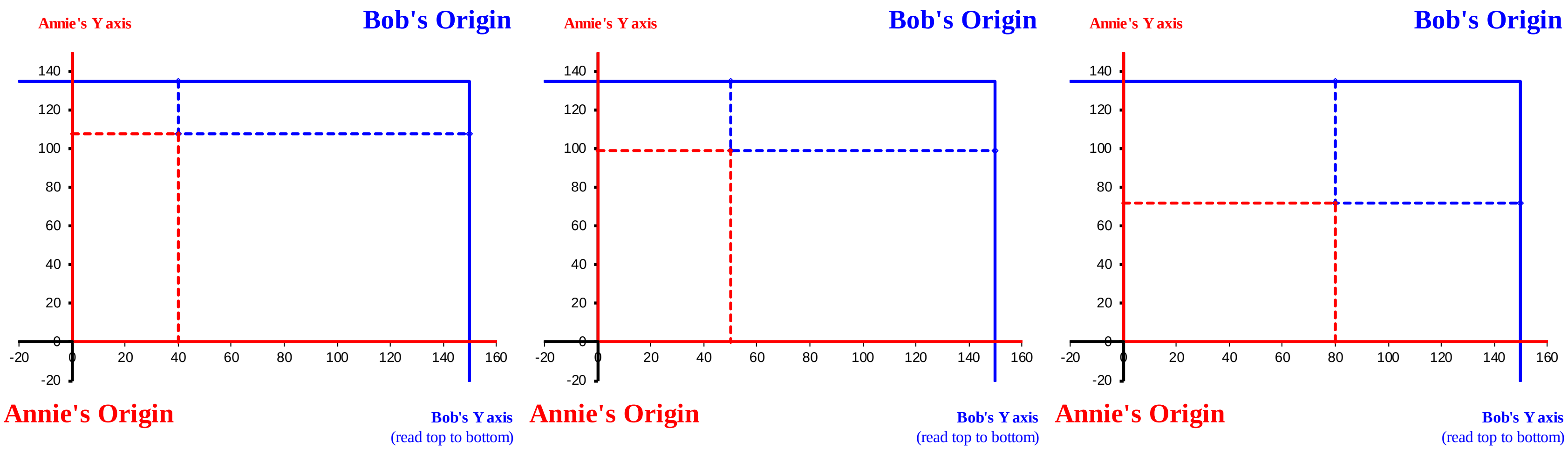
☑ Initial Endowment		MRS
X	Y	(= Y/X)
110	27	0.25
☑ Bob's barter bundle		
70.0	63.0	0.90

The gummi:chocolate exchange rate of 9:10 was used because it represents the relative scarcity of the two candies (135 gummies and 150 chocolates total). Other exchange rates will lead to different Pareto optimal outcomes. One can think of the bargaining game as being one of choosing the rate at which gummis are exchanged for chocolate. The outcome depends on Annie's and Bob's relative bargaining abilities, a topic explored in greater detail in questions 10-14. One can derive bounds on these Pareto optimal outcomes but this is easier to do if we use an Edgeworth box.

The above analysis is tractable but a bit difficult to follow because you must move back and forth between two diagrams (and keep track of how MRS changes in the tables). A barter Edgeworth box provides the SAME analysis in a single elegant diagram. An Edgeworth box includes information from both individuals by superimposing one individual's information on top of the other's in a way that automatically keeps track of trading between them. As Annie and Bob exchange chocolate and gummis with each other, the total goods in their "economy" remains fixed: there are a total of 150 packages of chocolate and 135 packages of chewy candy (gummis).[1] If one were to create a 150×135 box, every point in that box would represent a different distribution of chocolates and gummis for Annie and Bob. If we use Annie's axis as the bottom left of our box and we use the point (150,135) as the upper right of the box, then this point becomes Bob's origin with Bob turned "upside down." This is easiest to see (and easiest to draw) by actually turning your book (or notebook) upside down when analyzing or drawing in Bob's information. **Figure 5 Panels A-C** depict the distributions described in **Figures 2-4** and the **Distributions_(fig.5)** sheet allows you create ANY distribution you wish using sliders.

Figure 5

Panel A | Panel B | Panel C



Panel A:

	x	y
Annie's	40	108
Bob's	110	27
Total	150	135

Panel B:

	x	y
Annie's	50	99
Bob's	100	36
Total	150	135

Panel C:

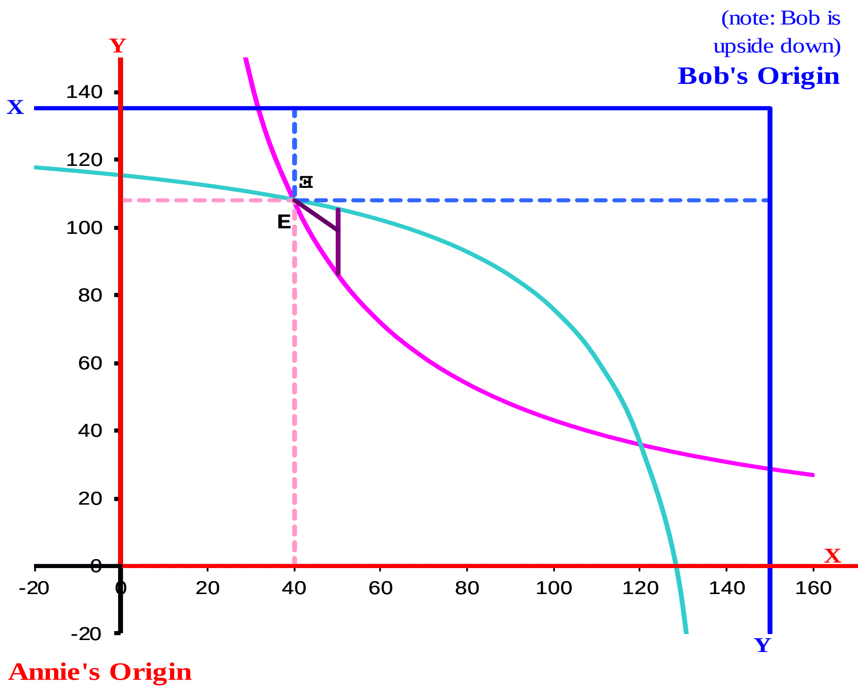
	x	y
Annie's	80	72
Bob's	70	63
Total	150	135

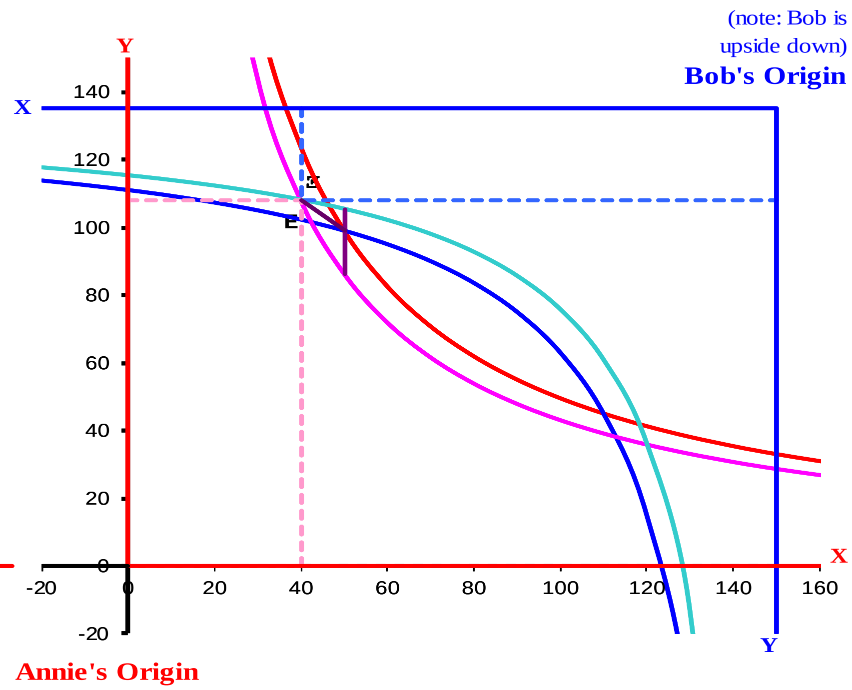
[1] Of course, this analysis ignores the main reason they went out trick-or-treating in the first place! As they eat the candy they reduce the goods available in their barter economy. For simplicity we assume that they exhibit a level of will power rarely seen in children – no eating occurs until they are done trading.

The genius of the Edgeworth Box is that Pareto superior and Pareto optimal distributions are quite simple to see once you include indifference curves. **Figure 6** depicts the same information provided in **Figures 2** and **3** above. Focus initially on **Panel 6A**: the indifference curves going through the endowment point create a "football" shaped area. This area is the set of distributions that are Pareto superior to the endowment point, E. A Pareto superior football will exist whenever MRSs are not equal across individuals. It is worth pointing out that putting Bob upside down does **NOT** change the interpretation of his MRS – his diagram has been turned 180º – a turn that does not affect the orientation of horizontal and vertical axes. Bob's MRS at his endowment point is less than Annie's MRS, the slopes of their respective indifference curves at their endowment point reflect this fact. Annie's indifference curve is steeper than Bob's at their (common) endowment point, **E**.

The vertical line between their indifference curves in **Figure 6A** represent the set of Pareto superior redistributions where Annie obtains 10 chocolates from Bob. The top point of that line segment represents a distribution in which Annie is the dominant bargainer since Bob remains indifferent between this bundle and his initial distribution. The bottom point of that line segment represents a distribution in which Bob is the dominant bargainer since Annie remains indifferent between this bundle and her initial distribution. The diagonal line between the endowment point and the point in the "middle" of that vertical segment represents the 9:10 trade discussed in **Figure 3**. This is not the end of trading because Annie and Bob do not have equal MRSs at the new distribution. This is easy to see in **Panel 6B**. There is a new smaller Pareto superior football through the 9:10 trade point. Such Pareto superior footballs will exist whenever MRSs are not equal. Trades will continue until a Pareto optimal outcome, a "zero football," is found.

Panel A **Figure 6 (Edgeworth Box versions of Figure 2 and 3)** **Panel B**





The Pareto optimal solution described in **Figure 4** in which Annie trades 36 gummies to Bob for 40 chocolates is shown as **Figure 7A**. The resulting distribution of goods, **M**, is a Pareto optimum since $MRS^A = MRS^B$. As argued above, this is only one of the Pareto optimal distributions achievable given the initial endowment. The green diagonal line in **Panel 7B** depicts the set of **all** Pareto optimal distributions. (One could alternatively call this the set of all tangencies between indifference curves, points of equal MRS, or zero footballs in the Edgeworth box.) Economists often call the set of Pareto optimal distributions the **contract curve**. (Note: this curve is linear because each has identical preferences.)

Definition: The **contract curve** is the set of all Pareto optimal distributions of goods.

Not all Pareto optimal distributions are achievable because either Annie or Bob would be better off remaining at their endowment point. In particular, points on the green diagonal below Annie's pink initial indifference curve are Pareto optimal but make Annie worse off (between Annie's origin and point **B**) and points on the diagonal above Bob's light blue initial indifference curve are Pareto optimal but make Bob worse off (between point **A** and Bob's origin). Points on the green Pareto optimal diagonal **inside** the initial Pareto superior football represent the set of Pareto optimal redistributions that will ultimately result given Annie's and Bob's initial bundles (this is the line segment **AB** in **Figure 7B**). The actual distribution chosen depends on the relative bargaining strengths of the two parties involved in the bargaining.

Panel A **Figure 7 (Edgeworth Box version of Figure 4)** **Panel B**

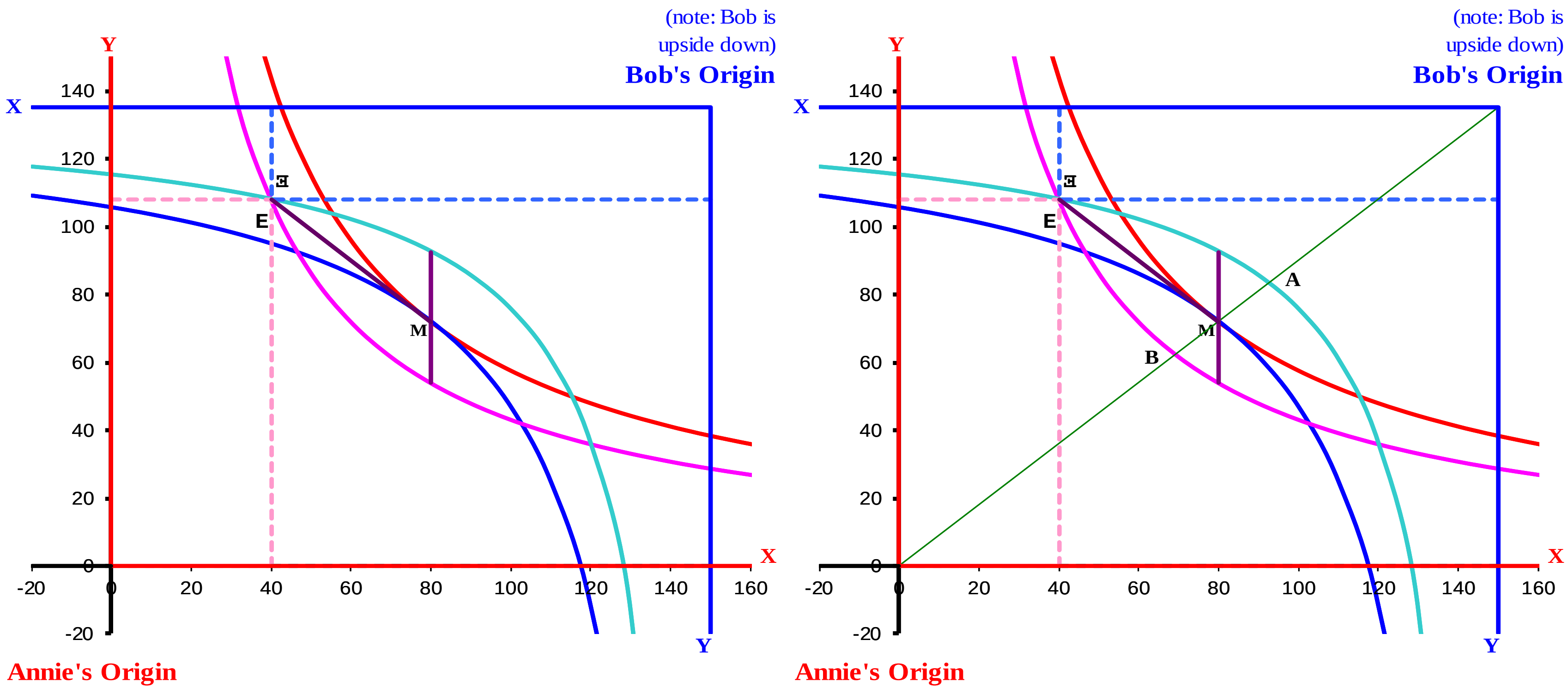
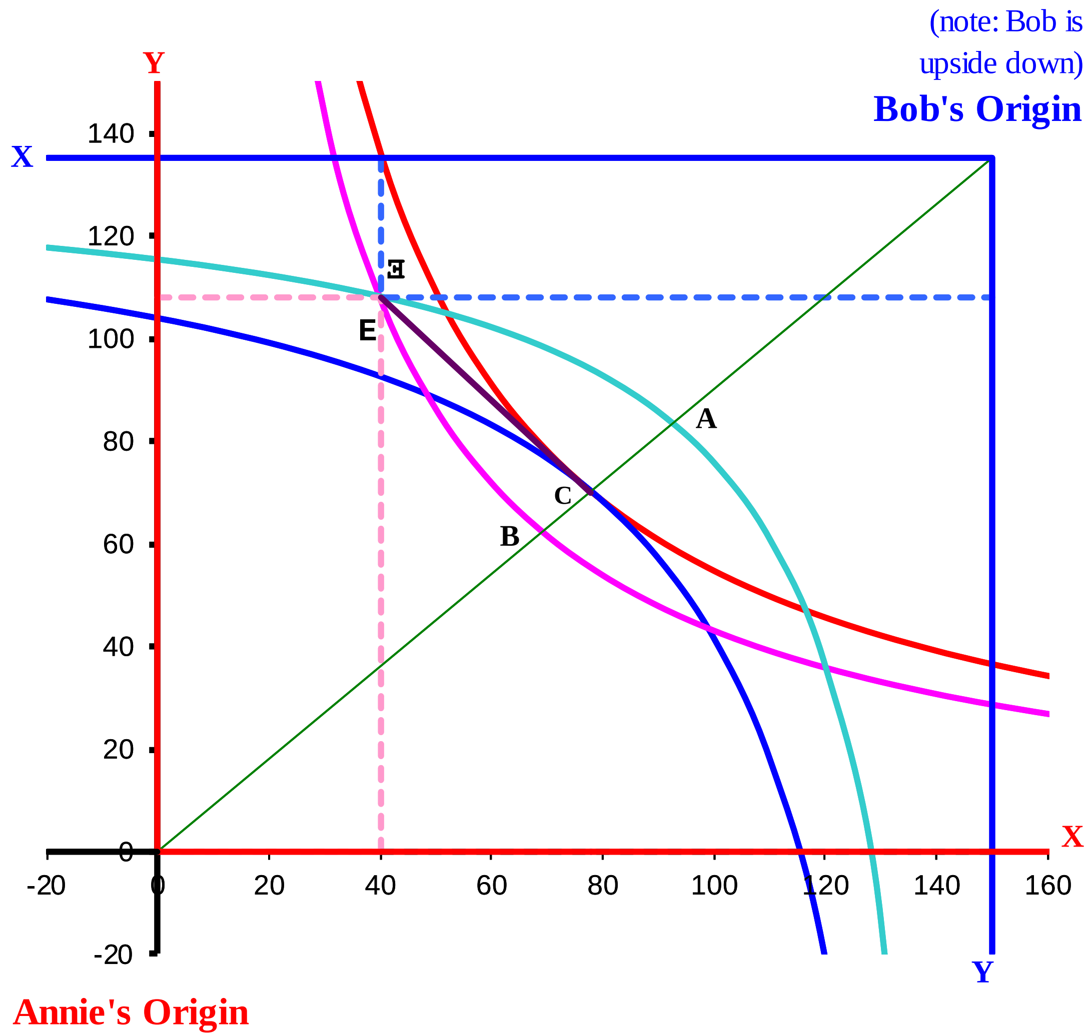


Figure 8 depicts one such alternative. This barter solution involves 1:1 trading in which Annie gives 38 gummis to Bob in exchange for 38 of his chocolates. This trade is shown as point **C**. Once again we have $MRS^A = MRS^B$ or a zero football as required by Pareto optimality. Relative to the scenario described above, this outcome is the result of Bob standing his ground and requiring a full gummi per chocolate (rather than 0.9 per chocolate).

You can probably envision Bob arguing with Annie that even though she is worse off than she was at **M** (since now she has $\mathbf{C}_{Annie}$ = (78,70) rather than $\mathbf{M}_{Annie}$ = (80,72)), she still has more of both candies than Bob (who ends up with with 6 fewer chocolates and five fewer gummies than Annie). This is an argument that may even sway their parents, should they be asked to enter into the fray.

	X	Y	MRS
Annie's ☑	78.0	70.0	0.90
Bob's ☑	72.0	65.0	0.90

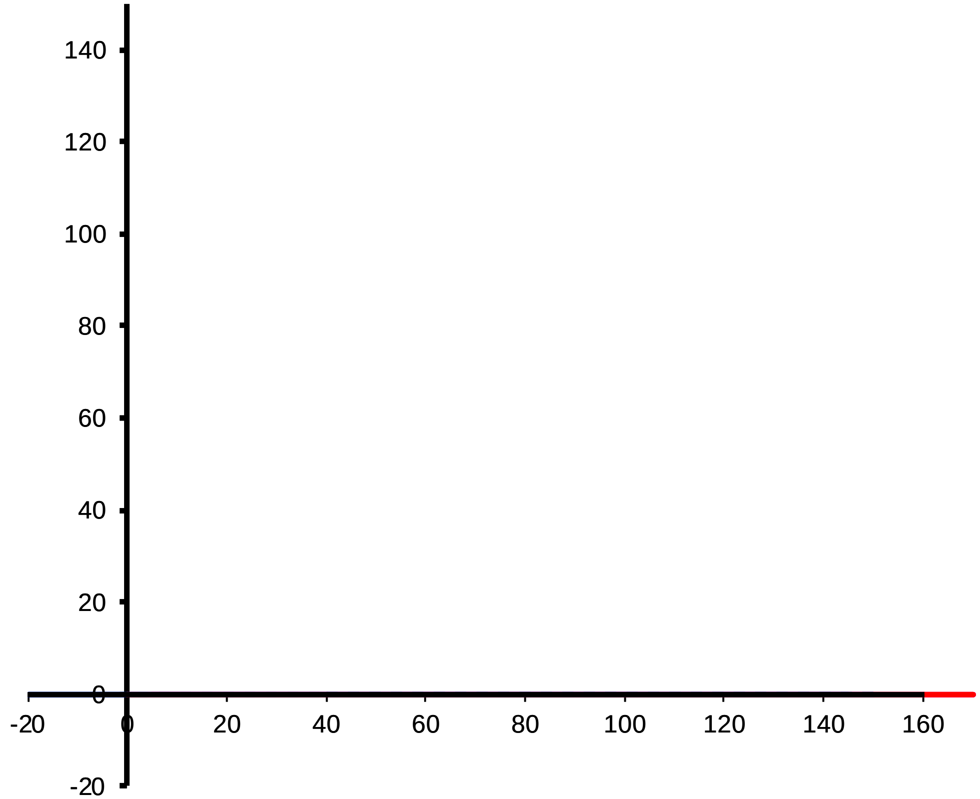
Figure 8



You can create your own figure using the sliders and clickboxes or use the **Scenarios** function on the **data** sheet to set up **Figures 6 - 8**.

Annie's		Bob's
☐	endowment as a vector	☐
☐	axes	☐
☐	E endowment Ǝ	☐
☐	Initial indifference curve	☐

	Endowment X	Endowment Y	MRS = Y/X
Annie's ☑	40	108	
Bob's ☑	110	27	
Total ☑	150	135	-



	X	Y	MRS
Annie's ☐			
Bob's ☐			

Unprotected calculating area

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click to go to **data** worksheet (to set up a figure from the text)

You can create your own figure using the sliders and clickboxes or use the **Scenarios** function on the **data** sheet to set up **Figures 6 - 8**.

Annie's		Bob's
☐	endowment as a vector	☐
☑	axes	☑
☑	E endowment Ǝ	☑
☑	Initial indifference curve	☑

	Endowment X	Endowment Y	MRS = Y/X
Annie's ☑	40	108	2.70
Bob's ☑	110	27	0.25
Total ☑	150	135	-

In this instance, (Annie,Bob) would like to trade some Y for more X?

A: ☑ Annie

Use the slider to represent transfering X between Bob and Annie.

X traded: More X to Bob More X to Annie

38.0

What is the smallest Y that Bob must be given to make this trade?

A: ☑ This is approximately -Bob's MRS times X traded: 14.3

What is the largest Y that Annie is willing to give to make this trade?

A: ☑ 52.6 Can such a trade occur in this instance?

A: ☑ Yes as long as 52.6 > 14.3

☐ Graphical representation of trade possibilities in this instance.

☑ Graphical representation of trade in this instance.

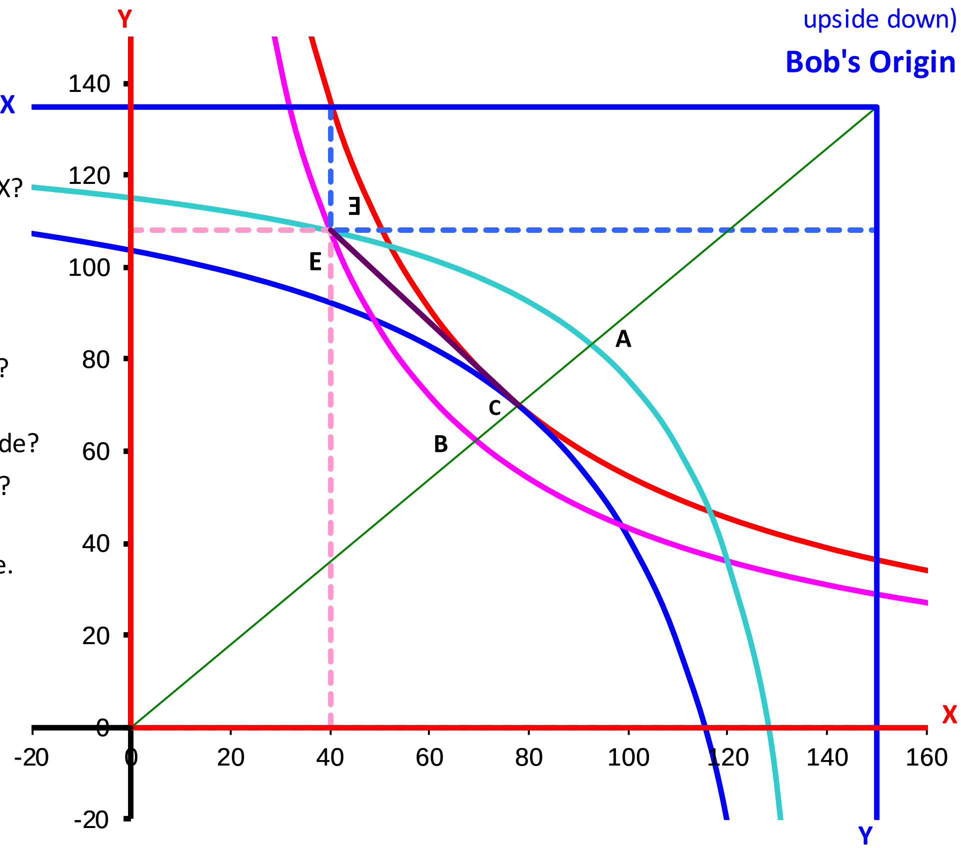
Annie is better at bargaining Bob is better at bargaining

Y traded: 38.0

Graphical representation of change in utility that results from this trade.

☑ For Annie ☑ For Bob

	X	Y	MRS
Annie's ☑	78.0	70.0	0.90
Bob's ☑	72.0	65.0	0.90



Are there further gains that could occur in this instance?

A: ☑ Yes as long as their MRSs at the new bundle are not equal.

☑ Contract curve

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Type = to start an equation.

click to go to **data** worksheet (to set up a figure from the text)

Glossary of terms used here (and some links to terminology sometimes seen in other treatments of the same topic).

Allocation of resources: An allocation of resources is a description of who gets what in an economy. In this instance, how are the candies distributed between Annie and Bob?

Barter: Exchanging goods for goods.

Bundle: A set of goods. Think of a shopping cart as a bundle, or in the present context, a trick-or-treat bag is a bundle of goods.

Contract curve: The set of efficient distributions of resources. This is alternatively called the set of Pareto optimal distributions of goods.

Convex preferences: This intuitively means that averages are weakly preferred to extremes. If you like two bundles equally well, then any bundle "between" these bundles you like at least as well as these bundles.

Distribution of goods: Used in the consumer context. It describes who gets what. How much does Annie have and how much does Bob have?

Economic agent: Economic term for any participant in an economy. In this case, Bob and Annie are economic agents.

Economic model: A simplified view of the world. In this context, there may well be more than two friends trading candies and there are certainly more than two candies, but we can obtain the essence of the situation by considering only two children and two candies.

Economic efficiency: Another term for Pareto efficiency.

Edgeworth box: An elegant way to show two consumers preferences and trading alternatives in a single graph.

Endowment: What you have initially. How much of each kind of candy each person has before they start trading.

Exchange: Another term for trade.

Goods: The things that consumers enjoy having and are able to trade with one another. Also called products.

Indifference curve: A way to graphically show the set of bundles that a person likes the same as one another.

Initial allocation is another name for endowment.

Initial distribution is another name for endowment.

Marginal Rate of Substitution: MRS is the rate at which you trade one good for another. In a graphical context it is minus the slope of the indifference curve. (The minus sign produces a positive number because, as long as preferences are well behaved, indifference curves are downward sloping.)

Monotonic preferences: more of a good makes you at least as happy as less of that good, holding all other goods constant.

MRS: Acronym for Marginal Rate of Substitution: The rate at which you trade one good for another. In a graphical context it is minus the slope of the indifference curve. (The minus sign produces a positive number because, as long as preferences are well behaved, indifference curves are downward sloping.)

Pareto criterion: Another way to say Pareto optimal. For example, you might say: This allocation satisfies the Pareto criterion.

Pareto efficient: Another way to say Pareto optimal.

Pareto improvement: Another way to say Pareto superior.

Pareto optimal: The general requirement for economic efficiency. An allocation of resources is Pareto optimal if you cannot reallocate resources in a way that makes one economic agent better off without making any other economic agent worse off. An alternative way to say an allocation is Pareto optimal is to say that there are no Pareto superior reallocations.

Pareto superior: A reallocation of resources that makes one party better off without making another party worse off. In this context, a redistribution of candies.

Preferences: The primitive notion in consumer theory. When faced with two bundles of goods A and B, a person can always say I prefer A to B, or B to A or I am indifferent between A and B. This most basically tells us what people like.

Trade ratio: The rate at which one good is traded for another. IF 2 ys are traded for each x then the trade ratio is 2 or 2:1.
Strictly convex preferences: This intuitively means that averages are preferred to extremes. If you like two bundles equally well, then any bundle "between" these bundles you like more than the endpoint bundles.
Well-behaved preferences: Monotonic and convex preferences. MRS declines (or remains constant) as x increases along the negatively sloped demand curve for well-behaved preferences.

Answers

1 Yes, trade ratio between 1 and 2 y per unit of x.
2 Yes, trade ratio between 3/4 and 1 y per unit of x.
3 Yes, trade ratio between 0.8 and 4/3 y per unit of x.
4 No, both have MRS = 0.75

5 No, any 1:1 trade will make Bob worse off in this instance since his MRS = 1 at his endowment point.
6 No, any 1:1 trade will make Bob worse off in this instance since his MRS = 1 at his endowment point.
7 Yes, Annie will give y to Bob for some of his x. Indeed, in this instance a trade of 2.5 for 2.5 is Pareto optimal.
8 No, Annie and Bob are already at a Pareto optimal allocation. No Pareto superior reallocations exist.

9	A)	7/5	Ytotal/Xtotal
	B)	6/7	Ytotal/Xtotal
	C)	1	Ytotal/Xtotal
	D)	3/4	Ytotal/Xtotal

The Excel figure shows that any exchange rate between 0.47:1 and 1.56:1 will produce a Pareto optimal outcome. Lower exchange rates are associated with Annie being a better bargainer, and higher exchange rates are associated with Bob being a better bargainer.

10 Annie obtains the entire gains from trade she is able to obtain 52.6 chocolates from Bob in exchange for 24.7 gummis. The resulting redistribution ((92.6, 83.3) for Annie and (57.4, 51.7) for Bob) is Pareto optimal since both individuals have the same MRS (of 0.90). This is point **A**.
11 Bob obtains the entire gains from trade when he is able to obtain 45.7 gummis from Annie in exchange for 29.3 chocolates. The resulting redistribution (of (69.3, 62.3) for Annie and (80.7, 72.7) for Bob) is Pareto optimal since both individuals have the same MRS (of 0.90). This is point **B**.
12 No. Annie would have wanted to stop at a 34 for 34 trade. This would have given her 74 of each good. She would have an MRS of 1 and not wish to trade further in this instance. Bob, on the other hand would wish to trade 41.5 in order to get to 68.5 of each good.
13 There is excess demand for y and excess supply of good y. The answer to this question provides a natural spring-board to a discussion of the allocating role of price.
14 No, the solution is not optimal. MRS = 1 for Annie and MRS = 0.80 for Bob.
15 Suppose $A = (x_A, y_A)$ and $B = (x_B, y_B)$ represents a Pareto optimal allocation of resources.

Then, Demand = Supply for chocolate implies $x_A + x_B = 150$, $x_B = 150 - x_A$

Similarly, Demand = Supply for gummis implies $y_A + y_B = 135$, $y_B = 135 - y_A$

Pareto optimal requires equal MRSs, $MRS_A = MRS_B$

$$MRS_A = y_A / x_A \text{ and } MRS_B = y_B / x_B.$$

Substituting the above equations relating x_A and x_B and y_A and y_B we obtain:

$$y_A / x_A = (135 - y_A)/(150 - x_A).$$

Cross multiplying, $150y_A - y_A x_A = 135x_A - y_A x_A$.

But we know that $MRS_A(\mathbf{A}) = y_A / x_A$.

Substituting we have $MRS_A(\mathbf{A}) = ((9/10)x_A)/x_A = 9/10 = 0.9$.

Since $MRS_A(\mathbf{A}) = MRS_B(\mathbf{B})$ at a Pareto optimum we know the same must be true for Bob.

17 In this instance, we know that trade occurs on the line through the endowment point having slope -1 and it is on the ray y = (9/10)x.

The equation for the line through the endowment point is: $y - 108 = -1\cdot(x - 40)$

Regrouping we obtain: $y = 108 - x + 40 = 148 - x$

Setting the two equations equal to one another yields $(9/10)x = 148 - x$

Simplifying: $9x = 1480 - 10x$

Solving for x: $19x = 1480$

$x = 1480/19 = 77\ 17/19 = 77.89$

The actual trade is therefore 37.89 or 37 17/19 rather than 38 to achieve a Pareto optimal outcome.

18 Annie: gives 30 y to Bob in exchange for 15 x from Bob.

Annie's final allocation		Bob's final allocation	
x = 55	y = 110	x = 45	y = 90
2 MRS Annie		2 MRS Bob	

The trade ratio of 2 y per x is reasonable given a total of y = 200 and x = 100.

Since Annie has more of both goods, she is better off than Bob in this instance.